Technical Notes 31C - Structural Design of Semicircular Brick Masonry Arches
Rev [Feb. 1971] (Reissued August 1986)

INTRODUCTION

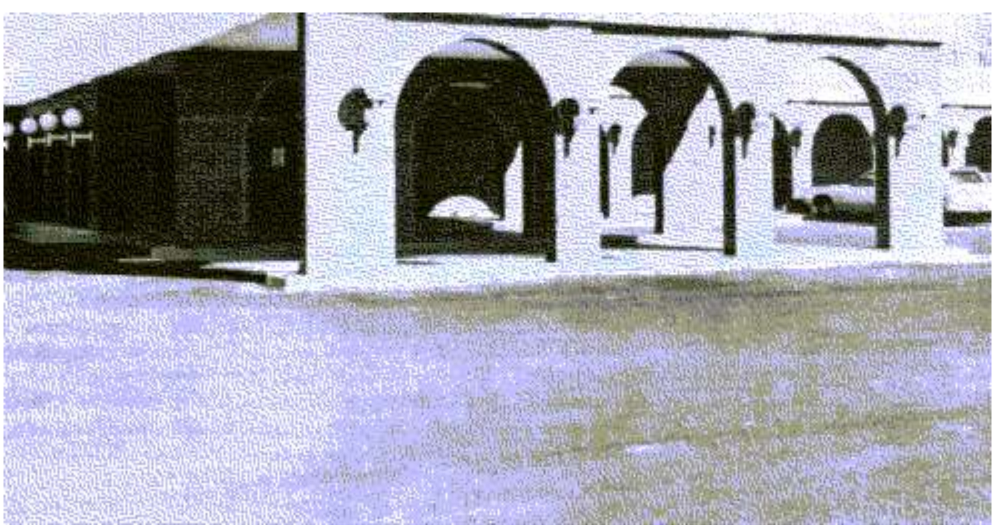
The semicircular arch is among the most popular arch forms used by architects today. Its smooth, continuous curve makes it easily adaptable to many architectural styles and applications.

This issue of *Technical Notes* presents recommended procedures and tables for the structural design of non-reinforced semicircular and segmental arches. *Technical Notes* 31 and 31A contain further information about general arch forms and their design.

DESIGN ASSUMPTIONS

Since the brick masonry arch is usually an integral part of a wall and not free-standing, basic design assumptions are made which assist in making the analysis.

The spring line is assumed to be located on a horizontal line one fourth the span length above the horizontal axis. The arches are assumed to be fully restrained at the spring line and the portion of semicircular arch above this line is analyzed in a manner similar to that for a parabolic arch, using the formulas in Section 10, Method A from *Frames and Arches*, by Valerian Leontovich, M.S., McGraw-Hill 1959.



Holiday Inn, McLean, Virginia

NOMENCLATURE

d = arch ring depth, in inches,

f = rise of arch, in feet,

f_m = allowable compressive stress, in psi,

H = horizontal thrust, in pounds,

H_{DL} = horizontal thrust, in pounds, caused by a uniform dead load,

L = span length, in feet,

n = number of shear planes (see *Technical Notes* 31A.)

P = allowable concentrated load, in pounds,

P' = maximum allowable concentrated load in pounds under combined loading provisions,

P* = additional concentrated load capacity caused by the uniform dead load.

t = wall thickness, in inches

v_m = allowable shear stress in brick masonry, in psi,

W = allowable uniform load, in pounds per foot,

x = length of wall required, in inches, to resist horizontal thrust

DEVELOPMENT OF TABLES

In the determination of the arch's capacity for uniform loads, the limiting factor was found to be the compressive strength of the brick masonry. Additional stresses due to the circular loading of the masonry above the intrados are also taken into consideration.

In determining the capacity for concentrated loads, the limiting factors were found to be bending at the center line of span, shear at the spring line (v_m = 40 psi), and maximum compressive stress (f_m). Tensile stresses were not permitted to develop at mid span.

Since axial forces develop in the arch ring from the concentrated and uniform loads, interaction formulas were developed for each loading condition. These formulas combine the axial stresses with the bending stresses.

ALLOWABLE LOADING

In all formulas used in this publication, d and t are measured in inches, and L is measured in feet. The following loading conditions were considered for analyzing a semicircular arch.

Uniform Load. Tables 1, 2, 3 and 4 give the allowable uniform loads occurring over the entire span length for a 1-in. thick arch ring. Figure 1 illustrates the following requirements and limitations:

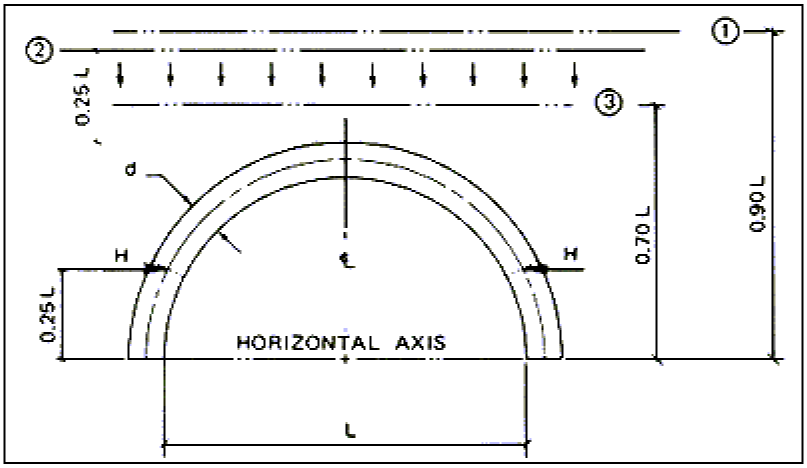


FIG. 1

1. Uniform load occurring between lines 1 and 3 (0.90 L and 0.70 L) are those provided for in the load tables.

2. Uniform loads occurring above line 1 may be ignored, at the discretion of the designer, provided arching action occurs in the brick masonry above the arch ring. (See discussion in *Technical Notes* 31 and 31A.)

3. There must be a minimum height of masonry (line 3) equal to 0.70 L above the horizontal axis. No superimposed loads are permitted below this line.

4. The maximum design height of masonry is 0.25 L above the crown for walls higher than line 2.

5. In all cases, the horizontal thrust (H) must be checked as shown in *Technical Notes* 31A, Fig. 4. For a given arch, the horizontal thrust is directly proportional to the uniform load.

6. The portion of wall that resists the horizontal thrust is assumed to be non-yielding to any lateral movement.

Concentrated Load*.* Table 5 gives the allowable concentrated loads occurring at the center line of span for a 1-in. thick arch ring. Figure 2 illustrates the following requirements and limitations:

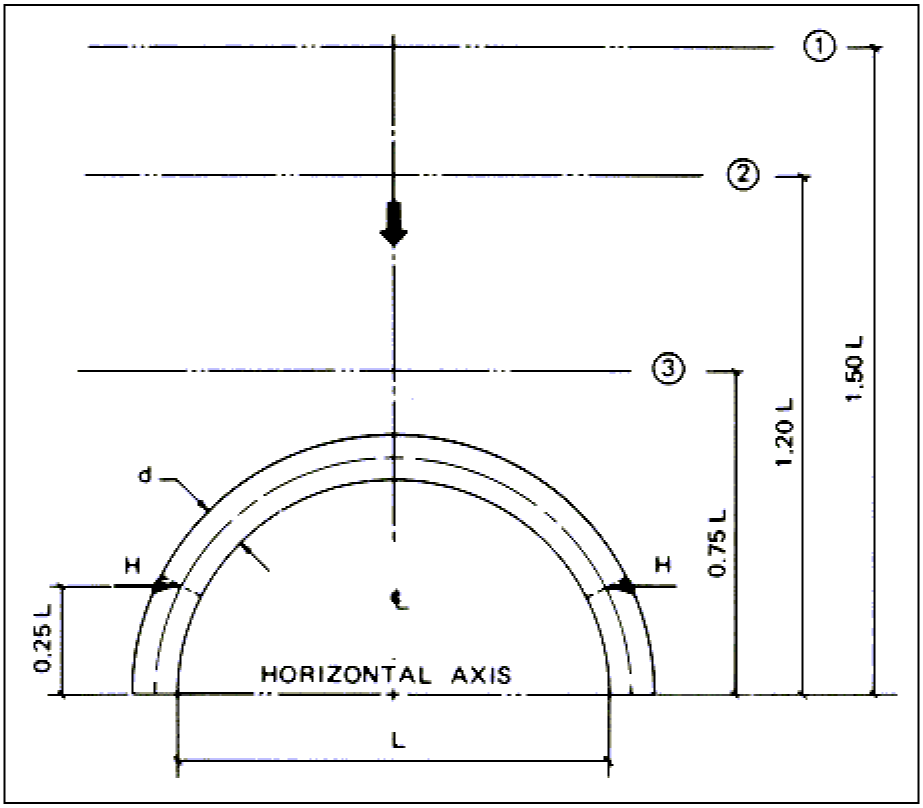


FIG. 2

1. Concentrated loads occurring between lines 2 and 3 (1.20 L and 0.75 L) are those provided for in the load table.

2. Concentrated loads occurring between lines 1 and 2 may be divided by the span length (L) and considered as equivalent uniform loads.

3. Concentrated loads occurring above line 1 (1.50 L) may be ignored, at the discretion of the designer, provided arching action occurs in the brick masonry above the arch ring. (See discussion in *Technical Notes* 31 and 31A.)

4. In all cases, condition 4 for uniform loads must be used with the resulting thrusts added to those of the concentrated loads.

5. There must be a minimum height of masonry (line 3) equal to 0.75 L above the horizontal axis. No superimposed loads are permitted below this line.

6. In all cases, the horizontal thrust H must be checked as shown in *Technical Notes* 31A, Fig. 4. For a given arch, the horizontal thrust is directly proportional to the concentrated load.

7. The portion of wall that resists the horizontal thrust is assumed to be non-yielding to any lateral movement.

TABLE 1
Allowable Uniform Load for f_m = 300 psi (t = 1 in.)

	d=3.5 in.		d=7.5 in.		d=11.5 in.		d=15.5 in.	
L	W	H	W	H	W	H	W	H
2	810	697	1520	1496	2071	2295	2509	3094
4	424	686	857	1489	1230	2289	1556	3089
6	277	659	591	1474	870	2277	1124	3078
8	193	611	444	1447	669	2257	875	3061
10	134	533	349	1406	538	2227	713	3036
12	86	420	280	1347	445	2185	597	3001
14	--	--	226	1268	374	2128	510	2956
16	--	--	180	1164	317	2055	440	2898
18	--	--	141	1034	269	1964	383	2825
20	--	--	105	873	228	1852	335	2737

TABLE 2
Allowable Uniform Load for f_m = 400 psi (t = 1 in.)

	d=3.5 in.		d=7.5 in.		d=11.5 in.		d=15.5 in.	
L	W	H	W	H	W	H	W	H
2	1082	930	2028	1996	2762	3061	3347	4127
4	569	919	1145	1989	1642	3055	2077	4121
6	376	892	792	1973	1164	3043	1501	4111
8	268	844	599	1947	897	3023	1172	4094
10	195	766	474	1906	724	2993	957	4068
12	137	653	385	1847	602	2951	804	4034
14	87	498	317	1767	510	2894	690	3988
16	--	--	261	1664	437	2821	600	3930
18	--	--	212	1533	377	2730	526	3858
20	--	--	170	1373	325	2618	464	3770

TABLE 3
Allowable Uniform Load for f_m = 500 psi (t = 1 in.)

	d=3.5 in.		d=7.5 in.		d=11.5 in.		d=15.5 in.	
L	W	H	W	H	W	H	W	H
2	1353	1163	2536	2495	3453	3827	4185	5159
4	714	1152	1434	2488	2055	3821	2597	5154
6	475	1125	993	2473	1458	3809	1879	5143
8	343	1077	753	2446	1125	3789	1468	5126
10	255	1000	600	2405	911	3759	1201	5101
12	188	886	491	2347	760	3717	1012	5066
14	131	731	408	2267	647	3660	870	5021
16	80	527	341	2164	558	3587	759	4962
18	--	--	284	2033	485	3496	669	4890
20	--	--	234	1872	423	3384	594	4802

TABLE 4
Allowable Uniform Load for f_m = 600 psi (t = 1 in.)

	d=3.5 in.		d=7.5 in.		d=11.5 in.		d=15.5 in.	
L	W	H	W	H	W	H	W	H
2	1624	1396	3044	2995	4145	4593	5023	6192
4	859	1385	1722	2988	2467	4587	3118	6186
6	573	1359	1194	2973	1752	4575	2257	6175
8	418	1310	908	2946	1353	4555	1765	6158
10	315	1233	725	2905	1097	4525	1445	6133
12	238	1120	596	2846	918	4483	1219	6099
14	174	964	499	2767	783	4426	1050	6053
16	118	760	421	2663	678	4353	918	5995
18	--	--	356	2532	592	4262	812	5922
20	--	--	299	2372	520	4150	723	5834

TABLE 5

Allowable Concentrated Load (t = 1 in.)*

L	d=3.5 in.		d=7.5 in.		d=11.5 in.		d=15.5 in.	
	P	H	P	H	P	H	P	H
2	15	12	703	547	1078	841	1451	1131
4	9	9	126	101	1075	841	1449	1131
6	13	16	38	35	418	332	1445	1132
8	20	25	40	41	97	86	967	764
10	28	36	48	52	88	84	199	172
12	38	49	58	67	93	95	162	150
14	48	65	70	83	104	111	160	156
16	61	83	84	102	117	130	168	171
18	75	103	100	124	133	152	181	191
20	90	125	117	148	151	177	198	215

TABLE 6

Maximum Concentrated Load Under Combined Loading Conditions (t= 1 in.)

L	f_m = 300 psi				f_m = 400 psi				f_m = 500 psi				f_m = 600 psi				f_m = 300 to 600 psi			
	d = 3.5		d = 7.5		d = 3.5		d = 7.5		d = 3.5		d = 7.5		d = 3.5		d = 7.5		d = 11.5 in.		d = 15.5 in.	
	P'	H'	P'	H'	P'	H'	P'	H'	P'	H'	P'	H'	P'	H'	P'	H'	P'	H'	P'	H'
2	328	256	703	547	328	256	703	547	328	256	703	547	328	256	703	547	1078	841	1451	1131
4	326	256	701	547	326	256	701	547	326	256	701	547	326	256	701	547	1075	841	1449	1131
6	324	256	698	548	324	256	698	548	324	256	698	548	324	256	698	548	1072	842	1445	1132
8	273	203	695	549	321	257	695	549	321	257	695	549	321	257	695	549	1065	842	1442	1133
10	235	170	691	550	310	227	691	550	316	257	691	550	316	257	691	550	1060	842	1440	1134
12	211	143	685	551	275	194	685	551	310	258	685	551	310	258	685	551	1055	843	1436	1135
14	195	125	685	504	252	169	676	552	304	258	676	552	304	258	676	552	1050	843	1431	1136
16	187	110	630	453	237	149	670	553	287	188	670	553	297	258	670	553	1045	843	1422	1137
18	183	98	587	410	228	133	661	554	273	169	661	554	289	259	661	554	1040	842	1411	1137
20	184	88	555	374	225	120	723	505	266	151	652	555	280	259	652	555	1025	842	1400	1138

*Values may be linearly interpolated except where horizontal lines occur. At these lines, the allowable load is ((0.241)(L + 0.083d^3 + 0.134)(L + 0.083d)^2 * d)/(1.34(L + 0.083d) - 0.0778d)

or the value above the line whichever is smaller The horizontal: thrust is 0.778P + 0.134(L + 0.083d)

or value above the line whichever is smaller.

***Combined Loading**.* When the uniform loads are combined with concentrated loads, the concentrated load capacity of the arch ring increases. This additional capacity is due to the compressive stress from the uniform load equalizing the tensile bending stress at mid span due to the concentrated load (M/S = P/A). This additional capacity may be expressed by the following formula:

$$P^* = \frac{H_{DL} d}{1.34L}$$

The values of P' and H' in Table 6 are the allowable capacities governed by compression or shear. They should be used only as a check when combined loadings are used.

In all cases, the actual load must be less than P', and less than the allowable load, P, plus the additional capacity P*. The total horizontal thrust must be checked and should be less than the maximum allowable for a uniform load.

SEGMENTAL ARCHES

Any segmental arch with f / L' > 0.29 but < 0.50 can be considered as an equivalent semicircular arch as shown in Fig. 3. Twice the radius is the equivalent L for use with the tables.

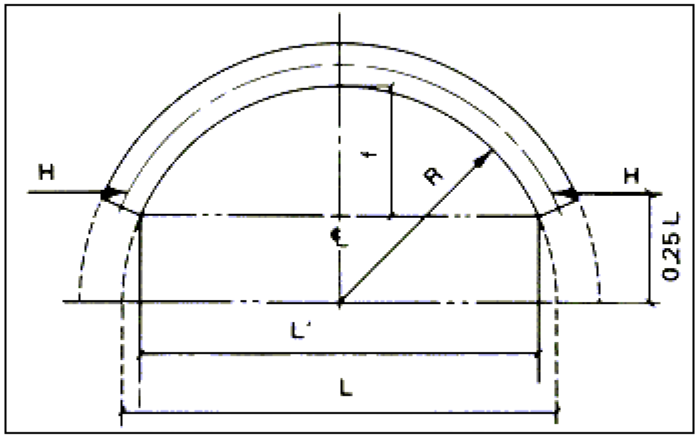


FIG. 3

ILLUSTRATIVE EXAMPLE

Design an arch to meet the requirements as shown in Fig. 4. The arch is semicircular; the horizontal axis is 6 ft above the base; the span, L, is 10 ft; the arch ring depth, d, is 12 in. (11 1/2 in. actual); and the nominal wall thickness, t, is 8 in. (7 1/2 in. actual). A beam reaction of 5000 lb is located at the center line of the span and 17 ft above the base. The uniform load consists of 1000 lb per ft dead load and 500 lb per ft live load occurring 14 ft above the base. Assume f_m = 400 psi and the brick masonry weighs 10 psf per 1-in. thickness.

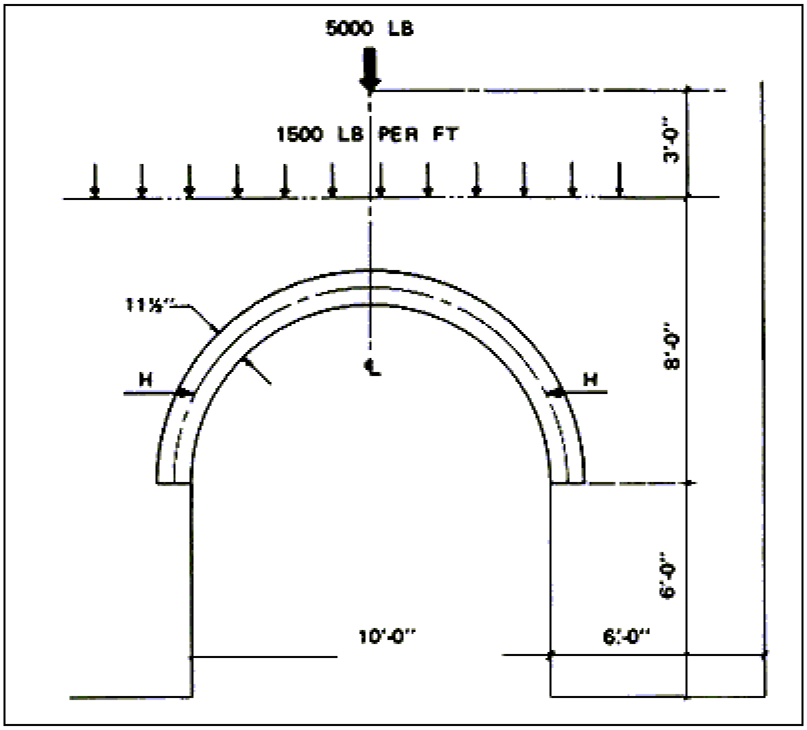


FIG. 4

Uniform Load

Wall DL =0.25 (10) (10) (7.5)	=	188 lb per ft
Arch DL = 1 (10) (7.5)	=	75
Floor DL	=	1000
Floor LL	=	500
Total Uniform Load	=	1763 lb per ft
Concentrated Load	=	5000 lb

$$\frac{DL}{TL} = \frac{1263}{1763} = 0.72$$

All the following calculations will be with 1 in. of wall thickness; actual t = 7.5 in., f_m = 400 psi, d = 11.5 in. and L= 10 ft.

Uniform Load

$$\text{The uniform load occurs at } \frac{8}{10} = 0.8L$$

Use Table 2, since 0.7L < 0.8L < 0.9L

From Table 2, W = 724 lb per ft and

H = 2993 lb

$$W\,(\text{actual}) = \frac{1763}{7.5} = 235 \text{ lb per ft } < 724 \qquad \text{O.K.}$$

$$H\,(\text{actual}) = \frac{235}{724}(2993) = 970 \text{ lb}$$

Concentrated Load

$$\text{The concentrated load occurs at } \frac{8+3}{10} = 1.1L$$

Use Table 5 since 0.75L < 1.1 L < 1.2L

From Table 5, P = 88 lb and H = 84 lb

$$P\,(\text{actual}) = \frac{5000}{7.5} = 667 \text{ lb } > 88 \qquad \text{N.G.}$$

However, since there is combined loading, advantage can be taken of the increased capacity due to the uniform load.

$$\therefore P^* = \frac{970(0.72)(11.5)}{1.34(10)} = 600 \text{ lb}$$

$$P\ (\text{allowable}) = 600 + 88 = 688\ \text{lb}$$

667 < 688 O.K.

From Table 6, P'= 1060 lb

667 < 1060 O.K.

$$H\ (\text{actual}) = \frac{667}{88}(84) = 636\ \text{lb}$$

Horizontal Thrust

H(total) =970 + 636 = 1606 < 2993 O.K.

At this point the wall shear caused by the horizontal thrust at the spring line should be checked. Assume V_m = 40 psi and n = 2

$$x = \frac{H}{v_m nt} = \frac{1606(7.5)}{40(2)(7.5)} = 20.5\,\text{in} < 6\text{ft} \qquad \text{O.K.}$$

The overturning moment of the support due to horizontal thrust should be checked next (see *Technical Notes* 31A). In this example, the horizontal thrust is 1606 (7.5) (8.5) = 102,000 ft-lb.

The resistance to overturning is a function of the overall axial load, wall shape, and reinforcement, if any. This is a separate analysis that should be performed after considering the total loading conditions on the entire structure.

CONCLUSION

This issue of *Technical Notes* has presented a simplified but conservative approach to a complex structural design problem. To provide an analysis for all possible assumptions and loading conditions is beyond the scope of this publication. Most loading conditions encountered will be similar to those in Fig. 1 and Fig. 2. To load an arch unsymmetrically defeats its use as a natural load-carrying structure and induces bending stresses that may cause failure.

If arches are to be loaded unsymmetrically or do not comply with the assumptions and limitations given in this *Technical Notes,* consideration should be given to reinforced brick masonry. (See *Technical Notes* 17A Revised, "Reinforced Brick Masonry - Flexural Design", and 17M, "Reinforced Brick Masonry Girders - Examples".) If conditions exist other than those covered in the tables, special analysis should be made by the designer.

The Brick Industry Association can*not* assume responsibility for the results obtained when using this *Technical Notes* Issue. It is beyond the scope of the Institute to anticipate every design situation that may arise. However, so long as the design criteria agree with the assumptions and limitations, satisfactory results can be obtained which will save countless hours of calculation time.